

Investigation of the field-induced ferromagnetic phase transition in spin polarized neutron matter: a lowest order constrained variational approach

G.H. Bordbar^{1,2 *}, Z. Rezaei¹ and Afshin Montakhab¹

¹*Department of Physics,
Shiraz University, Shiraz 71454, Iran[†],*

and

²*Research Institute for Astronomy and Astrophysics of Maragha,
P.O. Box 55134-441, Maragha, Iran*

Abstract

In this paper, the lowest order constrained variational (LOCV) method has been used to investigate the magnetic properties of spin polarized neutron matter in the presence of strong magnetic field at zero temperature employing AV_{18} potential. Our results indicate that a ferromagnetic phase transition is induced by a strong magnetic field with strength greater than 10^{18} G, leading to a partial spin polarization of the neutron matter. It is also shown that the equation of state of neutron matter in the presence of magnetic field is stiffer than the case in absence of magnetic field.

* Corresponding author. E-mail: bordbar@physics.susc.ac.ir

[†] Permanent address

I. INTRODUCTION

The magnetic field of neutron stars most probably originates from the compression of magnetic flux inherited from the progenitor star [1]. Using this point of view, Woltjer has predicted a magnetic field strength of order 10^{15} G for the neutron stars [2]. The field can be distorted or amplified by some mixture of convection, differential rotation and magnetic instabilities [3, 4]. The relative importance of these ingredients depend on the initial field strength and rotation rate of the star. For both convection and differential rotation, the field and its supporting currents are not likely to be confined to the solid crust of the star, but distributed in most of the stellar interior which is mostly a fluid mixture of neutrons, protons, electrons, and other more exotic particles [1]. Thompson et al. [5] argued that the newborn neutron stars probably combine vigorous convection and differential rotation making it likely that a dynamo process might operate in them. They expected fields up to $10^{15} - 10^{16} \text{ G}$ in neutron stars with few millisecond initial periods. On the other hand, according to the scalar virial theorem which is based on Newtonian gravity, the magnetic field strength is allowed by values up to 10^{18} G in the interior of a magnetar [6]. However, general relativity predicts the allowed maximum value of neutron star magnetic field to be about $10^{18} - 10^{20} \text{ G}$ [7]. By comparing with the observational data, Yuan et al. [8] obtained a magnetic field strength of order 10^{19} G for the neutron stars.

The strong magnetic field could have important influences on the interior matter of a neutron star. Many works have dealt with study of the magnetic properties and equation of state of the neutron star matter [9–20] and quark star matter [21–26] in the presence of strong magnetic fields. Some authors have considered the influence of strong magnetic fields on the neutron star matter within the mean field approximation [9, 12]. Yuan et al. [9] using the nonlinear $\sigma - \omega$ model, showed that the equation of state of neutron star matter becomes softer as the magnetic field increases. Also, Broderick et al. [10] employing a field theoretical approach in which the baryons interact via the exchange of $\sigma - \omega - \rho$ mesons, observed that the softening of the equation of state caused by Landau quantization is overwhelmed by stiffening due to the incorporation of the anomalous magnetic moments of the nucleons. It has been shown that the strong magnetic field shifts β -equilibrium and increases the proton fraction in the neutron star matter [10–12]. Yue et al. [13] have studied the neutron star matter in the presence of strong magnetic field using the quark-meson coupling (QMC)

model. Their results indicate that the Landau quantization of charged particles causes a softening in the equation of state, whereas the inclusion of nucleon anomalous magnetic moments lead to a stiffer equation of state. The effects of the magnetic field on the neutron star structure, through its influence on the metric has been studied by Cardall et al. [27]. Their results show that the maximum mass, in a static configuration for neutron star with magnetic field, is larger than the maximum mass obtained by uniform rotation. Through a field theoretical approach (at the mean field level) in which the baryons interact via the exchange of $\sigma - \omega - \rho$ mesons, Broderick et al. [14] have considered the effects of magnetic field on the equation of state of dense baryonic matter in which hyperons are present. They found that when the hyperons appear, the pressure becomes smaller than the case of pure nucleonic matter for all fields. Within a relativistic Hartree approach in the linear $\sigma - \omega - \rho$ model, the effects of magnetic field on cold symmetric nuclear matter and the nuclear matter in β -equilibrium have been investigated by Chakrabarty et al. [15]. Their results suggest that the neutron star mass is practically insensitive to the effects of the magnetic fields, whereas the radius decreases in intense fields .

In some studies, the neutron star matter was approximated by a pure neutron matter. Isayev et al. [16] considered the neutron matter in a strong magnetic field with the Skyrme effective interaction and analyzed the resultant self-consistent equations. They found that the thermodynamically stable branch extends from the very low densities to the high density region where the spin polarization parameter is saturated, and neutrons become totally spin polarized. Perez-Garcia et al. [18–20] studied the effects of a strong magnetic field on the pure neutron matter with effective nuclear forces within the framework of the non-relativistic Hartree-Fock approximation. They showed that in the Skyrme model there is a ferromagnetic phase transition at $\rho \sim 4\rho_0$ ($\rho_0 = 0.16 fm^{-3}$ is the nuclear saturation density), whereas it is forbidden in the $D1P$ model [18]. Beside these, they found that the neutrino opacity of magnetized matter decreases compared to the nonmagnetized case for the magnetic field greater than $10^{17} G$ [19]. However, more realistically, for the problem of the neutron star matter in astrophysics context, it is necessary to consider the finite temperature [17, 18, 23, 24] and finite proton fraction effects [9–15]. Isayev et al. [17] have shown that the influence of finite temperatures on spin polarization remains moderate in the Skyrme model, at least up to temperatures relevant for protoneutron stars. It has been also shown that for $SLy4$ effective interaction, even small admixture of protons to neutron

matter leads to a considerable shift of the critical density of the spin instability to lower values. For *SkI5* force, however, a small admixture of protons to neutron matter does not considerably change the critical density of the spin instability and increases its value [28].

In our previous works, we have studied the spin polarized neutron matter [29], symmetric nuclear matter [30], asymmetric nuclear matter [31], and neutron star matter [31] at zero temperature using LOCV method with the realistic strong interaction in the absence of magnetic field. We have also investigated the thermodynamic properties of the spin polarized neutron matter [32], symmetric nuclear matter [33], and asymmetric nuclear matter [34] at finite temperature with no magnetic field. In the above calculations, our results do not show any spontaneous ferromagnetic phase transition for these systems. In the present work, we study the magnetic properties of spin polarized neutron matter at zero temperature in the presence of the strong magnetic field using LOCV technique employing AV_{18} potential.

II. LOCV FORMALISM FOR SPIN POLARIZED NEUTRON MATTER

We consider a pure homogeneous spin polarized neutron matter composed of the spin-up (+) and spin-down (−) neutrons. We denote the number densities of spin-up and spin-down neutrons by $\rho^{(+)}$ and $\rho^{(-)}$, respectively. We introduce the spin polarization parameter (δ) by

$$\delta = \frac{\rho^{(+)} - \rho^{(-)}}{\rho}, \quad (1)$$

where $-1 \leq \delta \leq 1$, and $\rho = \rho^{(+)} + \rho^{(-)}$ is the total density of system.

In order to calculate the energy of this system, we use LOCV method as follows: we consider a trial many-body wave function of the form

$$\psi = F\phi, \quad (2)$$

where ϕ is the uncorrelated ground-state wave function of N independent neutrons, and F is a proper N -body correlation function. Using Jastrow approximation [35], F can be replaced by

$$F = S \prod_{i>j} f(ij), \quad (3)$$

where S is a symmetrizing operator. We consider a cluster expansion of the energy functional up to the two-body term,

$$E([f]) = \frac{1}{N} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2. \quad (4)$$

Now, we calculate the energy per particle up to the two-body term for two cases in the absence and presence of the magnetic field in two separate sections.

A. Energy calculation for the spin polarized neutron matter in the absence of magnetic field

The one-body term E_1 for spin polarized neutron matter in the absence of magnetic field ($B = 0$) is given by

$$E_1^{(B=0)} = \sum_{i=+,-} \frac{3}{5} \frac{\hbar^2 k_F^{(i)^2}}{2m} \frac{\rho^{(i)}}{\rho}, \quad (5)$$

where $k_F^{(i)} = (6\pi^2 \rho^{(i)})^{\frac{1}{3}}$ is the Fermi momentum of a neutron with spin projection i .

The two-body energy E_2 is

$$E_2^{(B=0)} = \frac{1}{2N} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \quad (6)$$

where

$$\nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12).$$

In the above equation, $f(12)$ and $V(12)$ are the two-body correlation function and nuclear potential, respectively. In our calculations, we employ the AV_{18} two-body potential [36],

$$V(12) = \sum_{p=1}^{18} V^{(p)}(r_{12}) O_{12}^{(p)}. \quad (7)$$

where

$$\begin{aligned} O_{12}^{(p=1-18)} = & 1, \sigma_1 \cdot \sigma_2, \tau_1 \cdot \tau_2, (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2), S_{12}, S_{12}(\tau_1 \cdot \tau_2), \\ & \mathbf{L} \cdot \mathbf{S}, \mathbf{L} \cdot \mathbf{S}(\tau_1 \cdot \tau_2), \mathbf{L}^2, \mathbf{L}^2(\sigma_1 \cdot \sigma_2), \mathbf{L}^2(\tau_1 \cdot \tau_2), \mathbf{L}^2(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2), \\ & (\mathbf{L} \cdot \mathbf{S})^2, (\mathbf{L} \cdot \mathbf{S})^2(\tau_1 \cdot \tau_2), \mathbf{T}_{12}, (\sigma_1 \cdot \sigma_2) \mathbf{T}_{12}, S_{12} \mathbf{T}_{12}, (\tau_{z1} + \tau_{z2}). \end{aligned} \quad (8)$$

In the above equation,

$$S_{12} = [3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2]$$

is the tensor operator and

$$\mathbf{T}_{12} = [3(\tau_1 \cdot \hat{r})(\tau_2 \cdot \hat{r}) - \tau_1 \cdot \tau_2]$$

is the isotensor operator. The above 18 components of the AV_{18} two-body potential are denoted by the labels $c, \sigma, \tau, \sigma\tau, t, t\tau, ls, ls\tau, l2, l2\sigma, l2\tau, l2\sigma\tau, ls2, ls2\tau, T, \sigma T, tT$, and τz , respectively [36]. In the LOCV formalism, the two-body correlation function $f(12)$ is considered as follows [37],

$$f(12) = \sum_{k=1}^3 f^{(k)}(r_{12}) P_{12}^{(k)}, \quad (9)$$

where

$$P_{12}^{(k=1-3)} = \left(\frac{1}{4} - \frac{1}{4}O_{12}^{(2)}\right), \left(\frac{1}{2} + \frac{1}{6}O_{12}^{(2)} + \frac{1}{6}O_{12}^{(5)}\right), \left(\frac{1}{4} + \frac{1}{12}O_{12}^{(2)} - \frac{1}{6}O_{12}^{(5)}\right). \quad (10)$$

The operators $O_{12}^{(2)}$ and $O_{12}^{(5)}$ are given in Eq. (8). Using the above two-body correlation function and potential, after doing some algebra, we find the following equation for the two-body energy:

$$\begin{aligned} E_2^{(B=0)} = & \frac{2}{\pi^4 \rho} \left(\frac{\hbar^2}{2m} \right) \sum_{JLS S_z} \frac{(2J+1)}{2(2S+1)} [1 - (-1)^{L+S+1}] \left| \left\langle \frac{1}{2}\sigma_{z1} \frac{1}{2}\sigma_{z2} \mid SS_z \right\rangle \right|^2 \\ & \int dr \left\{ \left[f_\alpha^{(1)'}{}^2 a_\alpha^{(1)2}(k_f r) + \frac{2m}{\hbar^2} (\{V_c - 3V_\sigma + V_\tau - 3V_{\sigma\tau} + 2(V_T - 3V_{\sigma T}) - 2V_{\tau z}\} a_\alpha^{(1)2}(k_f r) \right. \right. \\ & + [V_{l2} - 3V_{l2\sigma} + V_{l2\tau} - 3V_{l2\sigma\tau}] c_\alpha^{(1)2}(k_f r) (f_\alpha^{(1)})^2 \Big] + \sum_{k=2,3} \left[f_\alpha^{(k)'}{}^2 a_\alpha^{(k)2}(k_f r) \right. \\ & + \frac{2m}{\hbar^2} (\{V_c + V_\sigma + V_\tau + V_{\sigma\tau} + (-6k+14)(V_{t\tau} + V_t) - (k-1)(V_{ls\tau} + V_{ls}) \\ & + 2[V_T + V_{\sigma T} + (-6k+14)V_{tT} - V_{\tau z}\}] a_\alpha^{(k)2}(k_f r) \\ & + [V_{l2} + V_{l2\sigma} + V_{l2\tau} + V_{l2\sigma\tau}] c_\alpha^{(k)2}(k_f r) + [V_{ls2} + V_{ls2\tau}] d_\alpha^{(k)2}(k_f r) f_\alpha^{(k)2} \Big] \\ & + \frac{2m}{\hbar^2} \{V_{ls} + V_{ls\tau} - 2(V_{l2} + V_{l2\sigma} + V_{l2\sigma\tau} + V_{l2\tau}) - 3(V_{ls2} + V_{ls2\tau})\} b_\alpha^2(k_f r) f_\alpha^{(2)} f_\alpha^{(3)} \\ & \left. + \frac{1}{r^2} (f_\alpha^{(2)} - f_\alpha^{(3)})^2 b_\alpha^2(k_f r) \right\}, \quad (11) \end{aligned}$$

where $\alpha = \{J, L, S, S_z\}$ and the coefficient $a_\alpha^{(1)2}$, etc., are defined as

$$a_\alpha^{(1)2}(x) = x^2 I_{L, S_z}(x), \quad (12)$$

$$a_\alpha^{(2)2}(x) = x^2 [\beta I_{J-1, S_z}(x) + \gamma I_{J+1, S_z}(x)], \quad (13)$$

$$a_\alpha^{(3)2}(x) = x^2 [\gamma I_{J-1, S_z}(x) + \beta I_{J+1, S_z}(x)], \quad (14)$$

$$b_\alpha^{(2)}(x) = x^2[\beta_{23}I_{J-1,S_z}(x) - \beta_{23}I_{J+1,S_z}(x)], \quad (15)$$

$$c_\alpha^{(1)^2}(x) = x^2\nu_1 I_{L,S_z}(x), \quad (16)$$

$$c_\alpha^{(2)^2}(x) = x^2[\eta_2 I_{J-1,S_z}(x) + \nu_2 I_{J+1,S_z}(x)], \quad (17)$$

$$c_\alpha^{(3)^2}(x) = x^2[\eta_3 I_{J-1,S_z}(x) + \nu_3 I_{J+1,S_z}(x)], \quad (18)$$

$$d_\alpha^{(2)^2}(x) = x^2[\xi_2 I_{J-1,S_z}(x) + \lambda_2 I_{J+1,S_z}(x)], \quad (19)$$

$$d_\alpha^{(3)^2}(x) = x^2[\xi_3 I_{J-1,S_z}(x) + \lambda_3 I_{J+1,S_z}(x)], \quad (20)$$

with

$$\beta = \frac{J+1}{2J+1}, \quad \gamma = \frac{J}{2J+1}, \quad \beta_{23} = \frac{2J(J+1)}{2J+1}, \quad (21)$$

$$\nu_1 = L(L+1), \quad \nu_2 = \frac{J^2(J+1)}{2J+1}, \quad \nu_3 = \frac{J^3 + 2J^2 + 3J + 2}{2J+1}, \quad (22)$$

$$\eta_2 = \frac{J(J^2 + 2J + 1)}{2J+1}, \quad \eta_3 = \frac{J(J^2 + J + 2)}{2J+1}, \quad (23)$$

$$\xi_2 = \frac{J^3 + 2J^2 + 2J + 1}{2J+1}, \quad \xi_3 = \frac{J(J^2 + J + 4)}{2J+1}, \quad (24)$$

$$\lambda_2 = \frac{J(J^2 + J + 1)}{2J+1}, \quad \lambda_3 = \frac{J^3 + 2J^2 + 5J + 4}{2J+1}, \quad (25)$$

and

$$I_{J,S_z}(x) = \int dq \, q^2 P_{S_z}(q) J_J^2(qx). \quad (26)$$

In the last equation $J_J(x)$ is the Bessel function and $P_{S_z}(q)$ is defined as

$$\begin{aligned} P_{S_z}(q) &= \frac{2}{3}\pi[(k_F^{\sigma_{z1}})^3 + (k_F^{\sigma_{z2}})^3 - \frac{3}{2}((k_F^{\sigma_{z1}})^2 + (k_F^{\sigma_{z2}})^2)q \\ &\quad - \frac{3}{16}((k_F^{\sigma_{z1}})^2 - (k_F^{\sigma_{z2}})^2)^2 q^{-1} + q^3] \end{aligned} \quad (27)$$

for $\frac{1}{2}|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}| < q < \frac{1}{2}|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}|$,

$$P_{S_z}(q) = \frac{4}{3}\pi \min((k_F^{\sigma_{z1}})^3, (k_F^{\sigma_{z2}})^3) \quad (28)$$

for $q < \frac{1}{2}|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}|$, and

$$P_{S_z}(q) = 0 \quad (29)$$

for $q > \frac{1}{2}|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}|$, where σ_{z1} or $\sigma_{z2} = +1, -1$ for spin up and down, respectively.

B. Energy calculation of spin polarized neutron matter in the presence of magnetic field

Now we consider the case in which the spin polarized neutron matter is under the influence of a strong magnetic field. Taking the uniform magnetic field along the z direction, $B = B\hat{k}$, the spin up and down particles correspond to parallel and antiparallel spins with respect to the magnetic field. Therefore, the contribution of magnetic energy of the neutron matter is

$$E_M = -M_z B, \quad (30)$$

where M_z is the magnetization of the neutron matter which is given by

$$M_z = N\mu_n\delta. \quad (31)$$

In the above equation, $\mu_n = -1.9130427(5)$ is the neutron magnetic moment (in units of the nuclear magneton). Consequently, the energy per particle up to the two-body term in the presence of magnetic field can be written as

$$E([f]) = E_1^{(B=0)} + E_2^{(B=0)} - \mu_n B\delta, \quad (32)$$

where $E_1^{(B=0)}$ and $E_2^{(B=0)}$ are given by Eqs. (5) and (11), respectively. It should be noted that in usual thermodynamic treatments the external magnetic field energy ($\frac{1}{8\pi} \int dV B^2$) is usually left out since it does not affect the thermodynamic properties of matter [38]. In fact the magnetic field energy arises only from the magnetostatic energy in the absence of matter, but we are interested in the contribution of internal energy which excludes the energy of magnetic field. Therefore, the magnetic field contribution, $E_{mag} = \frac{B^2}{8\pi}$, which is the *energy density* (or “magnetic pressure”) of the magnetic field in the absence of matter is usually omitted [16, 38].

Now, we minimize the two-body energy with respect to the variations in the function $f_\alpha^{(i)}$ subject to the normalization constraint [39],

$$\frac{1}{N} \sum_{ij} \langle ij | h_{S_z}^2 - f^2(12) | ij \rangle_a = 0, \quad (33)$$

where in the case of spin polarized neutron matter, the function $h_{S_z}(r)$ is defined as follows,

$$h_{S_z}(r) = \begin{cases} \left[1 - 9 \left(\frac{J_F^2(k_F^{(i)} r)}{k_F^{(i)} r} \right)^2 \right]^{-1/2} & ; \quad S_z = \pm 1 \\ 1 & ; \quad S_z = 0 \end{cases} \quad (34)$$

From minimization of the two-body cluster energy, we get a set of coupled and uncoupled differential equations which are the same as those presented in Ref. [39], with the coefficients replaced by those indicated in Eqs. (12)–(20). By solving these differential equations, we can obtain correlation functions to compute the two-body energy.

III. RESULTS AND DISCUSSION

Our results for the energy per particle of spin polarized neutron matter versus the spin polarization parameter for different values of the magnetic field at $\rho = 0.2 \text{ fm}^{-3}$ have been shown in Fig. 1. We have found that for the values of magnetic field below 10^{18} G , the corresponding energies of different magnetic fields are nearly identical. This shows that the effect of magnetic field below $B \sim 10^{18} \text{ G}$ is nearly insignificant. From Fig. 1, we can see that the spin polarization symmetry is broken when the magnetic field is present and a minimum appears at $-1 < \delta < 0$. By increasing the magnetic field strength from $B \sim 10^{18} \text{ G}$ to $B \sim 10^{19} \text{ G}$, the value of spin polarization corresponding to the minimum point approaches -1 . We also see that by increasing the magnetic field, the energy per particle at minimum point (ground state energy) decreases, leading to a more stable system. For each density, we have found that above a certain value of the magnetic field, the system reaches a saturation point and the minimum energy occurs at $\delta = -1$. For example at $\rho = 0.2 \text{ fm}^{-3}$, for $B \gtrsim 1.8 \times 10^{19} \text{ G}$, the minimum energy occurs at $\delta = -1$. However, this threshold value of the magnetic field increases by increasing the density. In Fig. 2, we have presented the ground state energy per particle of spin polarized neutron matter as a function of the density for different values of the magnetic field. For each value of the magnetic field, it is shown that the energy per particle increases monotonically by increasing the density. However, the increasing rate of energy versus density increases by increasing the magnetic field. This indicates that at higher magnetic fields, the increasing rate of the contribution of magnetic energy versus density is more than that at lower magnetic fields. In order to clarify this behavior, we have presented the energy contribution of spin polarized neutron matter up to the two-body term in the cluster expansion ($E_1 + E_2$), and the magnetic energy contribution (E_M) separately, as a function of density in Fig. 3. This figure shows that for the spin polarized neutron matter, the difference between the magnetic energy contributions (E_M) of different magnetic fields is substantially larger than that for the energy contribution

$(E_1 + E_2)$. Fig. 4 shows the ground state energy per particle of spin polarized neutron matter as a function of the magnetic field for different values of density. We can see that by increasing the magnetic field up to a value about 10^{18} G, the energy per particle slowly decreases, and then it rapidly decreases for the magnetic fields greater than this value. This indicates that above $B \sim 10^{18}$ G, the effect of magnetic field on the energy construction of the spin polarized neutron matter becomes more important.

In Fig. 5, the spin polarization parameter corresponding to the equilibrium state of the system is plotted as a function of density for different values of the magnetic field. It is seen that at each magnetic field, the magnitude of spin polarization parameter decreases by increasing the density. Fig. 5 also shows that for the magnetic fields below 10^{18} G, at high densities, the system nearly becomes unpolarized. However, for higher magnetic fields, the system has a substantial spin polarization, even at high densities. In Fig. 6, we have plotted the spin polarization parameter at the equilibrium as a function of the magnetic field at different values of density. This figure shows that below $B \sim 10^{18}$ G, no anomaly is observed and the neutron matter can only be partially polarized. This partial polarization is maximized at lower densities and amounts to about 14% of its maximum possible value of -1 . From Fig. 6, we can also see that below $B \sim 10^{17}$ G, the spin polarization parameter is nearly zero. This clearly confirms the absence of the magnetic ordering for the neutron matter up to $B \sim 10^{17}$ G. For the magnetic fields greater than about 10^{18} G, it is shown that the magnitude of spin polarization rapidly increases by increasing the magnetic field. This shows a ferromagnetic phase transition in the presence of a strong magnetic field. For each density, we can see that at high magnetic fields, the value of spin polarization parameter is close to -1 . The corresponding value of the magnetic field increases by increasing the density.

The magnetic susceptibility (χ) which characterizes the response of a system to the magnetic field, is defined by

$$\chi(\rho, B) = \left(\frac{\partial M_z(\rho, B)}{\partial B} \right)_\rho \quad (35)$$

In Fig. 7, we have plotted the ratio $\chi/N|\mu_n|$ for the spin polarized neutron matter versus the magnetic field at three different values of the density. As can be seen from Fig. 7, for each density, this ratio shows a maximum at a specific magnetic field. This result confirms the existence of the ferromagnetic phase transition induced by the magnetic field. We see

that the magnetic field at phase transition point, B_m , depends on the density of the system. Fig. 8 shows the phase diagram for the spin polarized neutron matter. We can see that by increasing the density, B_m grows monotonically. It explicitly means that at higher densities, the phase transition occurs at higher values of the magnetic field.

From the energy of spin polarized neutron matter, at each magnetic field, we can evaluate the corresponding pressure ($P_{kinetic}$) using the following relation,

$$P_{kinetic}(\rho, B) = \rho^2 \left(\frac{\partial E(\rho, B)}{\partial \rho} \right)_B \quad (36)$$

Our results for the kinetic pressure of spin polarized neutron matter versus the density for different values of the magnetic field have been shown in Fig. 9. It is obvious that with increasing the density, the difference between the pressure of spin polarized neutron matter at different magnetic field becomes more appreciable. Fig. 9 shows that the equation of state of the spin polarized neutron matter becomes stiffer as the magnetic field strength increases. This stiffening is due to the inclusion of neutron anomalous magnetic moments. This is in agreement with the results obtained in Refs. [10, 13]. It should be noted here that to find the total pressure related for the neutron star structure, the contribution from the magnetic field, $P_{mag} = \frac{B^2}{8\pi}$, should be added to the kinetic pressure [10, 14]. However, in this work we are not interested in the neutron star structure and have thus omitted the contribution of “magnetic pressure” in our calculations for neutron matter [16]. This term, if included, simply adds a constant amount to the curves depicted in Fig. 9.

IV. SUMMARY AND CONCLUDING REMARKS

We have recently calculated several properties of the spin polarized neutron matter in the absence of magnetic field using the lowest order constrained variational method with AV_{18} potential. In this work, we have generalized our calculations for spin polarized neutron matter in the presence of strong magnetic field at zero temperature using this method. We have found that the effect of magnetic fields below $B \sim 10^{18} G$ is almost negligible. It was shown that in the presence of magnetic field, the spin polarization symmetry is broken and the energy per particle shows a minimum at $-1 < \delta < 0$, depending on the strength of the magnetic field. We have shown that the ground state energy per particle decreases

by increasing the magnetic field. This leads to a more stable system. It is seen that the increasing rate of energy versus density increases by increasing the magnetic field. Our calculations show that above $B \sim 10^{18} \text{ G}$, the effect of magnetic field on the properties of neutron matter becomes more important. In the study of spin polarization parameter, we have shown that for a fixed magnetic field, the magnitude of spin polarization parameter at the minimum point of energy decreases with increasing density. At strong magnetic fields with strengths greater than 10^{18} G , our results show that a field-induced ferromagnetic phase transition occurs for the neutron matter. By investigating the magnetic susceptibility of the spin polarized neutron matter, it is clear that as the density increases, the phase transition occurs at higher values of the magnetic field. Through the calculation of pressure as a function of density at different values of the magnetic field, we observed the stiffening of the equation of state in the presence of the magnetic field.

Finally, we would like to address the question of thermodynamic stability of such neutron stars at ultra-high magnetic fields. One may wonder if the effect of magnetic pressure, $P_{mag} = \frac{B^2}{8\pi}$, which we have omitted here, is added to the kinetic pressure $P_{kinetic}$, then at ultra-strong magnetic fields, the system might become gravitationally unstable due to excessive outward pressure. For the fields considered in this work (up to 10^{20} G), this scenario does not seem likely [7]. We note that the increase of magnetic field leads to stiffening of the equation of state (Fig. 9) which in turn leads to larger mass and radius for the neutron star [40]. This in turn increases the effect of gravitational energy, offsetting the increased pressure. We also note that the existence of a well-defined thermodynamic energy minimum for all fields considered in our work indicates the thermodynamic stability of our system. The existence of such well-defined minimum energy is unaffected by the addition of magnetic energy. The detailed analysis of such situations along with accompanying change in proton fraction is a possible avenue for future research.

Acknowledgments

We would like to thank two anonymous referees for constructive criticisms. This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha. We

wish to thank Shiraz University Research Council.

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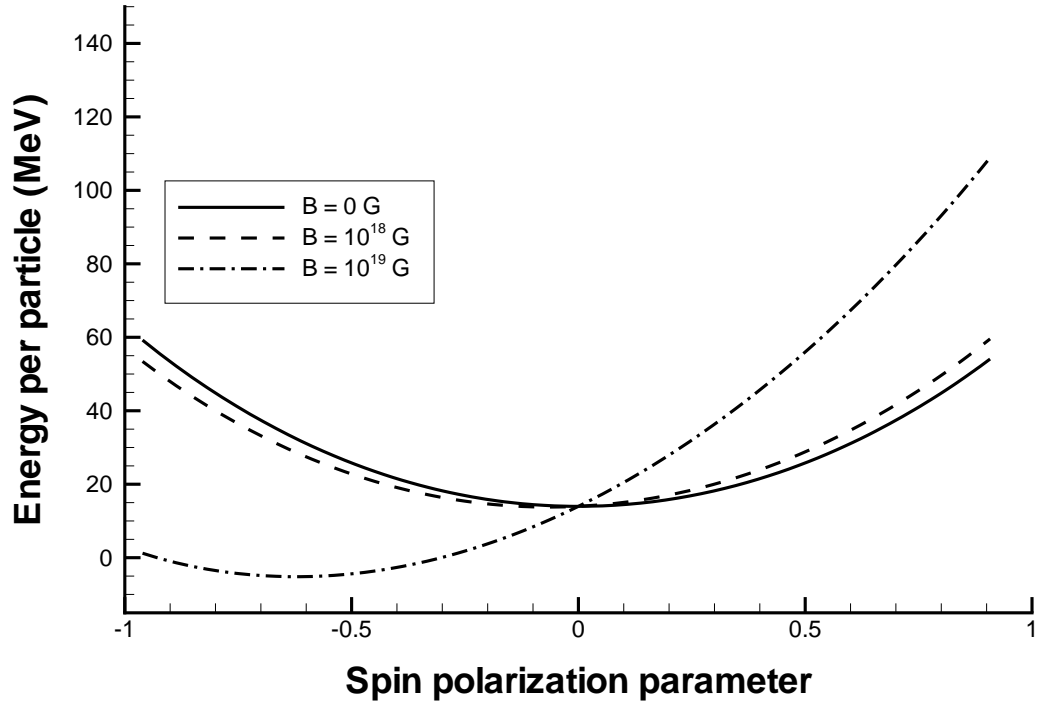


FIG. 1: The energy per particle versus the spin polarization parameter (δ) for different values of the magnetic field (B) at $\rho = 0.2 \text{ fm}^{-3}$.

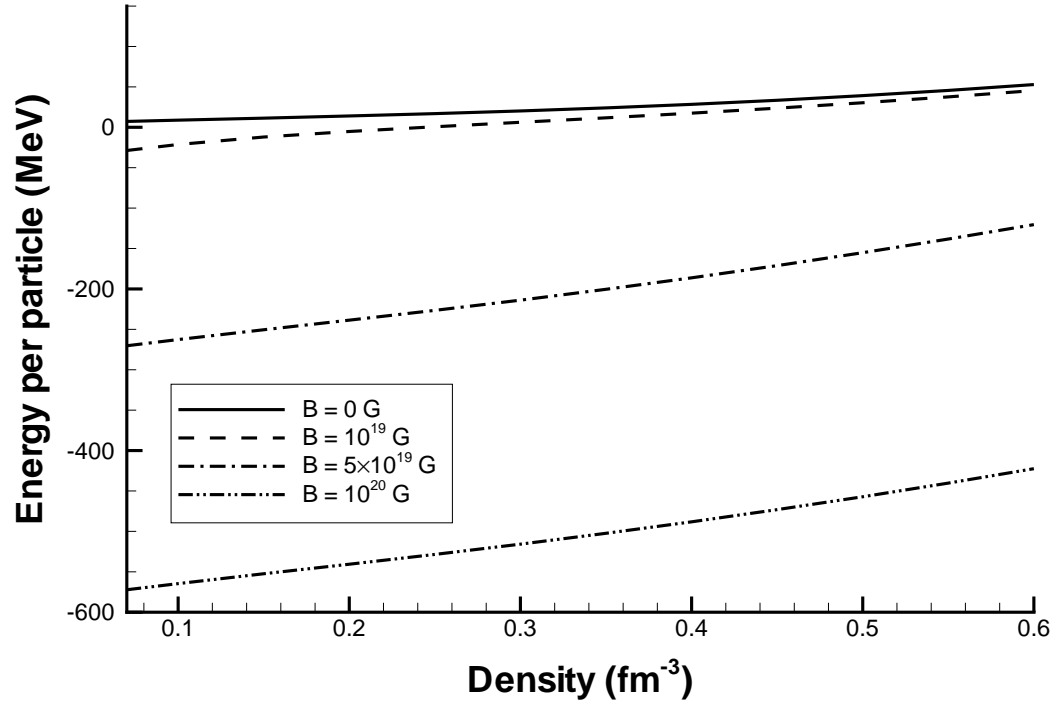


FIG. 2: The ground state energy per particle as a function of the density at different values of the magnetic field (B).

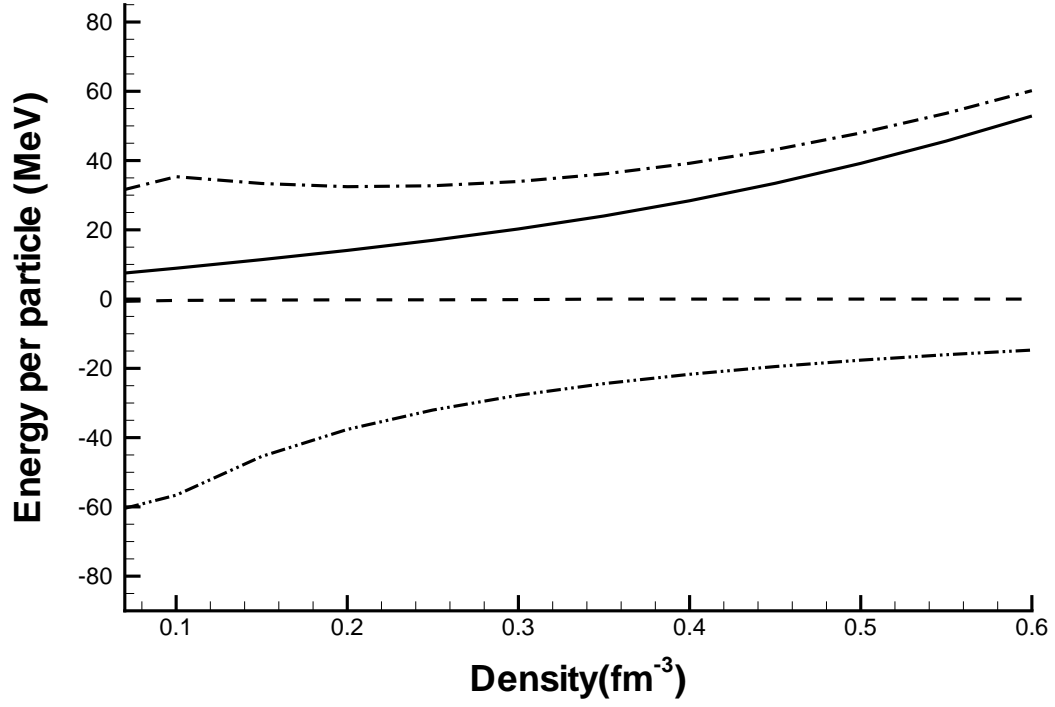


FIG. 3: The energy contribution of spin polarized neutron matter in the cluster expansion up to the two body term ($E_1 + E_2$) for the magnetic fields $B = 10^{18} \text{ G}$ (solid curve) and $B = 10^{19} \text{ G}$ (dashed dotted curve), and the contribution of magnetic energy (E_M) for magnetic fields $B = 10^{18} \text{ G}$ (dashed curve) and $B = 10^{19} \text{ G}$ (dashed dotted dotted curve).

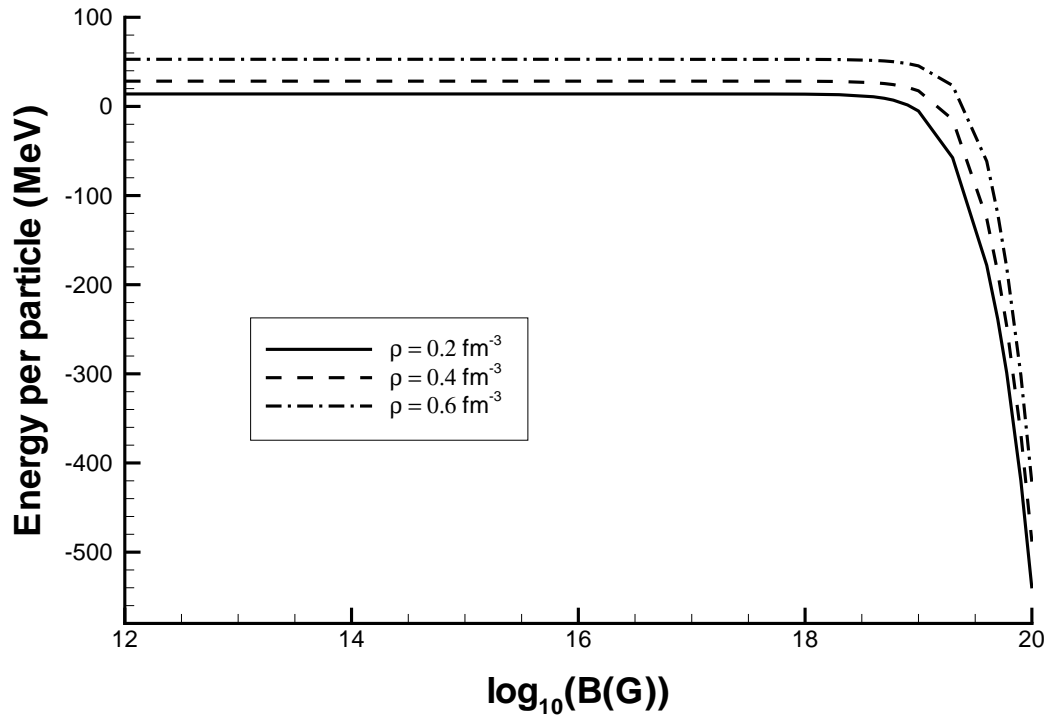


FIG. 4: The ground state energy per particle as a function of the magnetic field (B) at different values of the density (ρ).

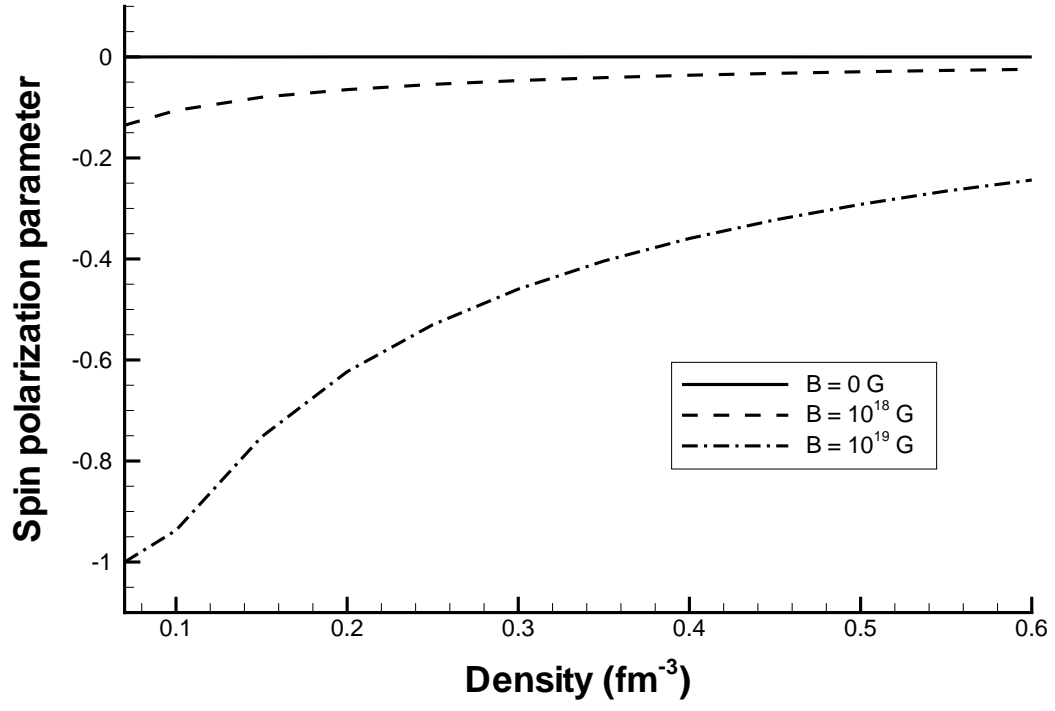


FIG. 5: The spin polarization parameter at the equilibrium state of the system as a function of the density at different values of the magnetic field (B).

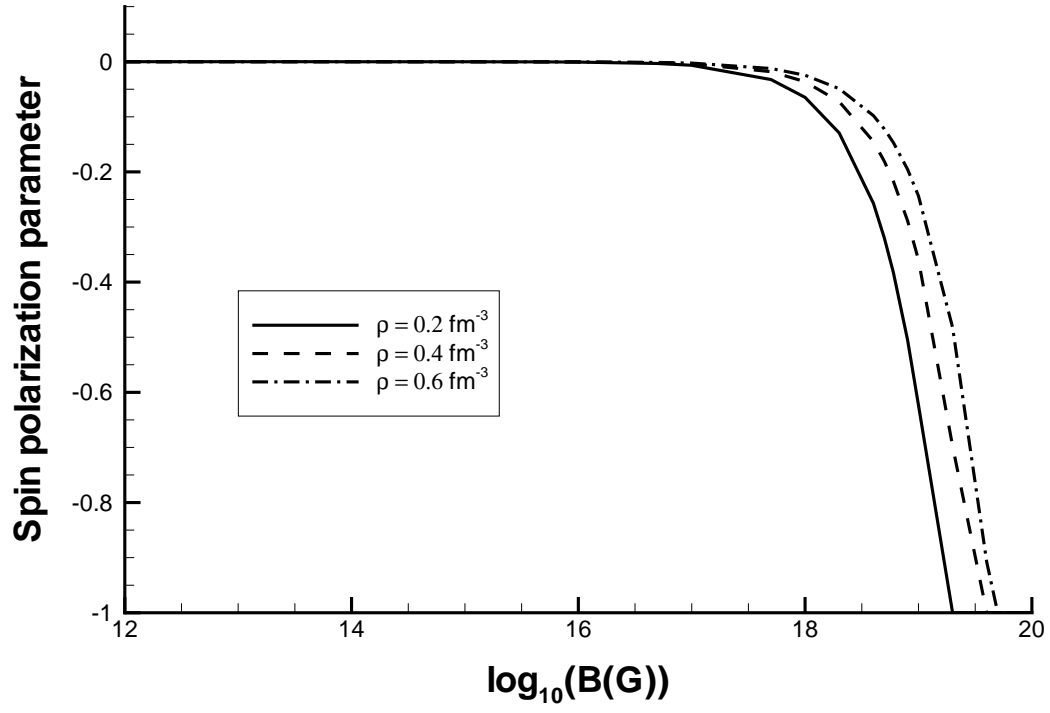


FIG. 6: The spin polarization parameter corresponding to the equilibrium state of the system as a function of the magnetic field (B) at different values of the density (ρ).

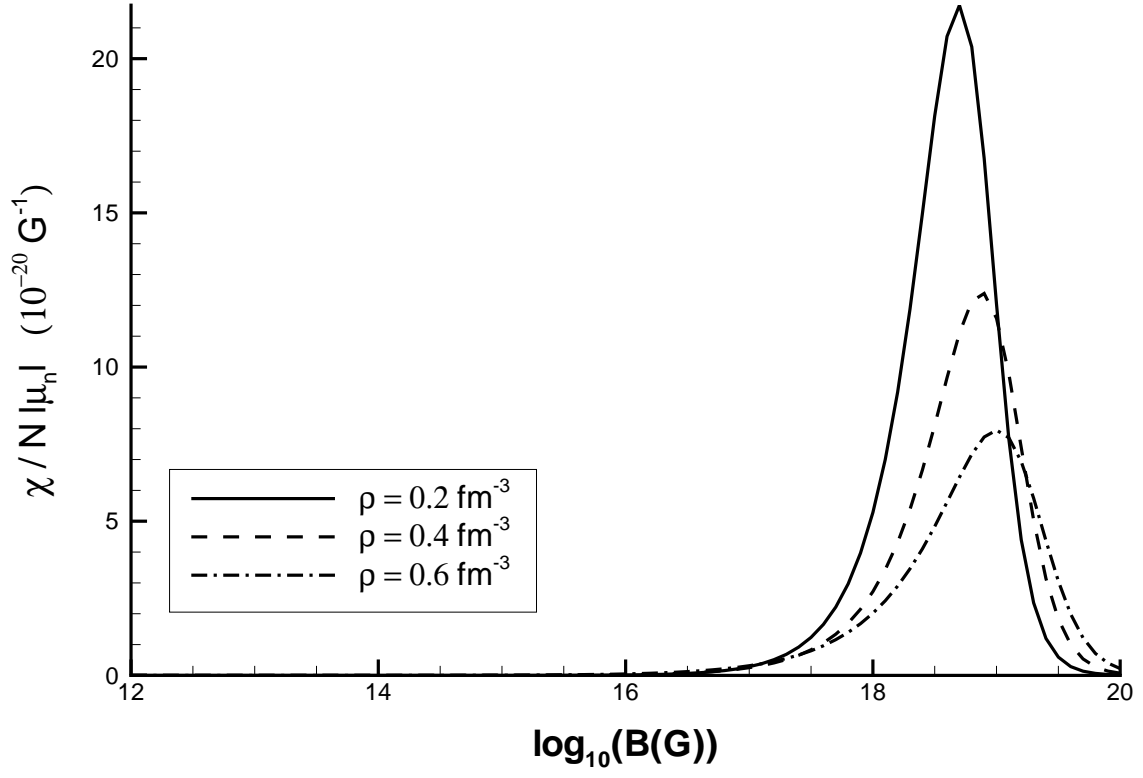


FIG. 7: The magnetic susceptibility ($\chi/N|\mu_n|$) as a function of the magnetic field (B) at different values of the density (ρ).

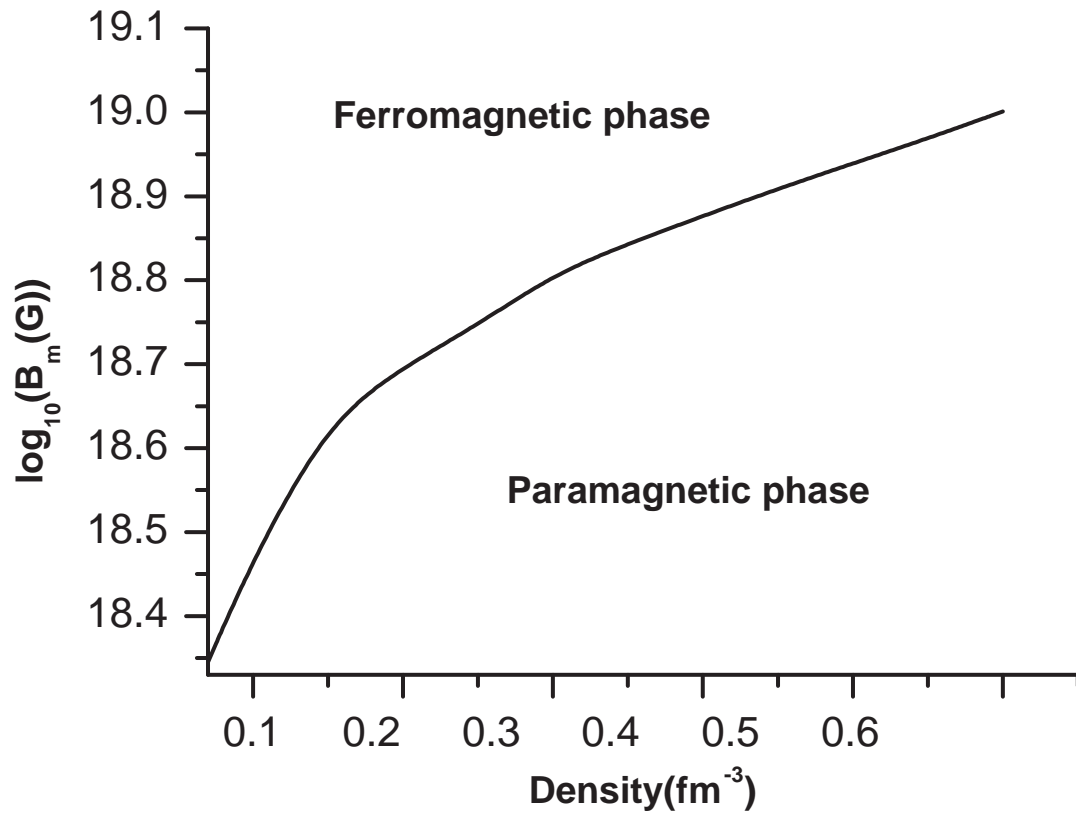


FIG. 8: Phase diagram for the spin polarized neutron matter in the presence of strong magnetic field.

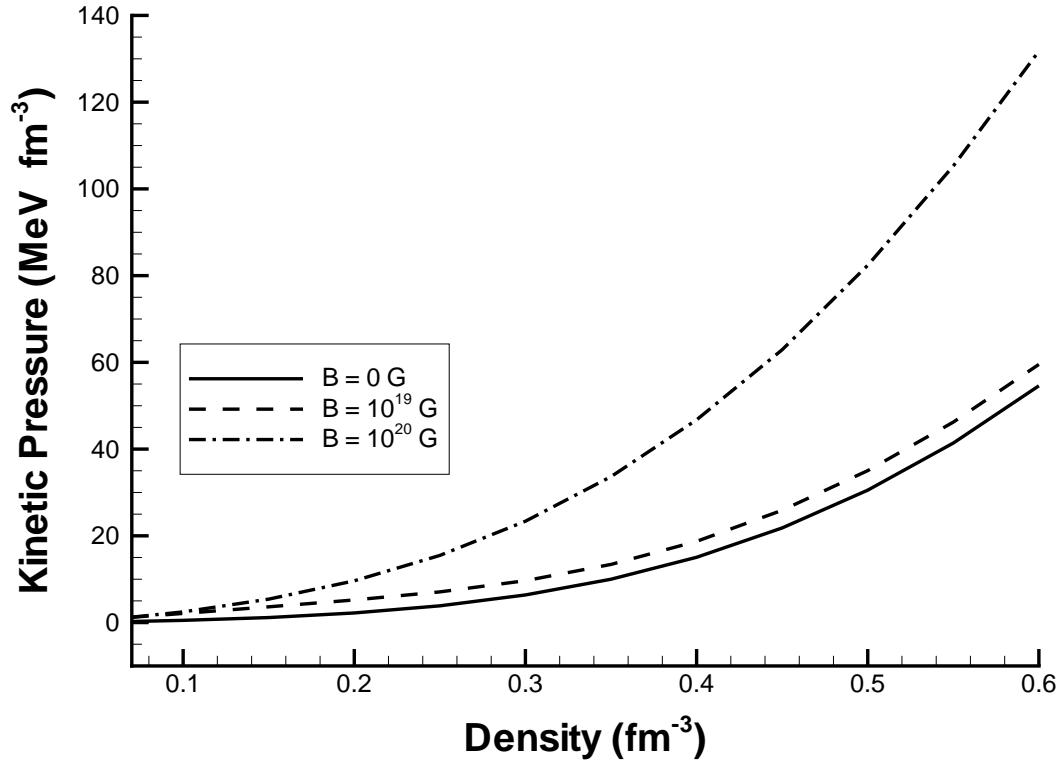


FIG. 9: The equation of state of spin polarized neutron matter for different values of the magnetic field (B).